1-2 Nim

Topics: logic, patterns, addition, counting, subtraction
Materials: Counters (tiles, beans, pennies, etc.) and/or paper and pencil
Time: 1-3 lessons

You can take one or two counters from the pile. How do you get the last one?

Why We Love 1-2 Nim

Nim is fun, challenging, and rewarding for a wide range of students. Completely unlocking the game is an exciting and powerful achievement for a student. Extensions for the game abound.

The Launch

It's good to highlight a few things when you launch 1-2 Nim. First, students will win and they will lose, and it's important to remember to do both gracefully. But second, losing is better than winning, in a way, because every time you lose, you can see what strategy your opponent used to beat you, and then learn that strategy. That's how you become a N*im Master*.

Launch the game by playing a few demonstration games with students.

Instructions

Nim is a two-player game. Start with a pile of 10 counters. On your turn, remove one or two counters from the pile. You must take at least one counter on your turn, but you may not take more than two. Whoever takes the last counter wins.

Example Game

Start with 10 counters in the pile.
Player A takes 2 counters, leaving 8.
Player B takes one counter, leaving 7.
Player A takes two counters, leaving 5.
Player B takes one counter, leaving 4.
Player A takes one counter, leaving 3.
Player B takes one counter, leaving 2.
Player A takes two counters, leaving 0 and winning the game.

Play several demonstration games as needed. When the students understand the rules, have them play against each other in pairs. Students can try to challenge the teacher if they think they have a strategy that can win.

The Work

As students play, the teacher can move among them and challenge them to play, or ask them what they've noticed so far. The teacher should be able to beat the student unless the student plays perfectly. When students have played for 10 - 15 minutes or so, bring them together again and discuss what they've noticed so far. Students may have noticed that when they can give their opponent 3 counters, for example, they win. (We call this the *3-trap*, since if you can give your opponent 3 counters, you have effectively trapped them, and can win the game no matter what they do.)

Pose the central question and discuss: how can you win at 1-2 Nim?

Students may have philosophical questions related to game-playing: what does it mean to have a winning position or losing position in a game? Is there luck in the game? Where does the game go from feeling "random" to feeling like you can control it?

These are productive conversations. Before students go back to work, there are two points to underline to make the exploration productive:

1) Making the game simpler makes it easier!
   How can you make the game simpler? By shrinking the pile. Challenge the students to play you with a pile of 1 counter. Do they want to go 1st or 2nd? What about with 2 counters? It's so easy it feels like a joke, but this is what serious mathematicians do, and we get valuable information here.

2) Make a table!
   This is how the data you collect by making the game easier can actually help you. The beginning of a table might look like this.

Number of Counters	Winning Strategy
1	Go first. Take 1.
2	Go first. Take 2.
3	Go second
4	...
5	...

If students want to master the game, all they have to do is extend the table. It tells them what to do.

Prompts and Questions

The Central Question: how can you win 1-2 Nim?

Good questions for the teacher to ask students:

- What move should I (the teacher) make?
- How did you/they/I win that game?
- What do you think your/my opponent will do if you/I take two counters?
- Would you like to take back your move?
- What have you noticed about this game?

Possible student conjectures (all interesting, all false or incomplete) that may arise:

- Whoever goes first wins.
- Whoever goes second wins.
- Odd vs. even numbers of beginning counters determines your strategy.
- It matters/doesn't matter what you do until there are less than six counters in the pile.
- Whoever can give their opponent four open counters wins.

The Wrap

It may be wise to end class without a conclusion, depending on where students are, and discuss again on a subsequent day. Send them home to try out the game against friends and family, and refine their strategy.

When students are ready, discuss strategy—do students have any ideas of how to win, regardless of the size of the pile? Once they share, ask if anyone would like to try another game against you. Let them get advice from their peers (students can quietly raise one or two fingers to suggest what they think should happen). Can they beat you?

The major breakthrough is that the table actually tells you what to do. Have a student share a table and discuss its contents. What patterns do students see? Students should be able to defend these results.

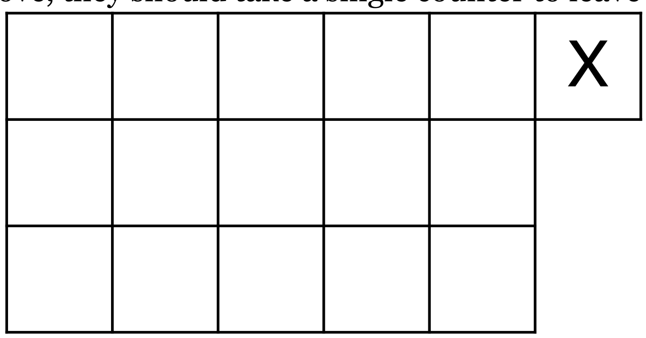
Number of Counters	Winning Strategy
1	Go first. Take 1.
2	Go first. Take 2.
3	Go second.
4	Go first. Take 1.
5	Go first. Take 2.
6	Go second.
7	Go first. Take 1.

You can play students with the table visible. On your turn, look at the table and talk out loud what you should do, i.e., “There are 7 counters, and its my turn. So the table says, take 1.” The takeaway here is that the table is literally instructions for winning.

Students will also, hopefully, notice the pattern in the table. (Go first, take 1. Go first, take 2. Go second.) It looks like the “3-trap” actually extends to a “6-trap” and a “9-trap,” and so on. In other words, the winning strategy might be as simple as: on your turn, give your opponent a multiple of three. Does that really work? Challenge the class to a game with 25 counters. Let students discuss their strategy, and then choose a student to play you. Can they win?

Finally, when students can articulate a winning strategy and successfully beat you, there’s still a question of *why* this “give your opponent a multiple of three” strategy succeeds. Here’s a sketch of an argument that shows why it does.

Arrange, let’s say, 16 counters in a 3 by 5 array, with one extra counter. When the first player makes their move, they should take a single counter to leave a 3 by 5 array.



Whatever their opponent does, they’ll always be able to give back a 3 by *something* array. Try it out! So the choice of arrangement of tiles actually makes the argument clear.

There’s a moral to this exploration: the way to *become* a master of nim, or any game, is to apply mathematical rigor and organization. In other words, thinking and working mathematically makes you powerful.

To end, you can challenge students with a couple of potential extensions.

Variations

1) What’s the right first move if you’re playing 1-2 Nim with 150 counters? What about 542 counters?
2) Try 1-2-3 Nim: players may take one, two, or three counters per turn. How do you win this game?
3) Try 1-2-3 Poison: Whoever takes the last counter <u>loses</u>.
4) What about 1-3-4 Nim? Players may take one, three, or four counters, but NOT 2.

Tips for the Classroom

1. Demonstrate the game with student volunteers for at least three games (or many more!), until you are certain everyone understands it and is excited to play.

2. When demonstrating 1-2 Nim, narrate the game out loud, using mathematical language, and leaving empty space for students to chime in: "My opponent just took 2 leaving… [wait for students] 5 in the pile. Who has advice for what I should do next?"

3. Remind students that they will lose many games as they play, and that every loss is an opportunity to learn. Can they steal the strategy of the person who just beat them? Point out how students are trying out new strategies as they play you in demonstration games.

4. As students play each other, circulate to see what strategies they are developing. Challenge them to play you, and see if they can beat you.

5. Encourage student conjectures, but do not call them as true or false. Challenge students to break their own conjectures.

6. A big moment in taking ownership of the game is to change the size of the pile. Making the pile smaller makes it easier to understand and win. Making it bigger makes it more challenging.

7. We use the term "3-trap" to describe how you trap your opponent by giving them a pile of three counters. Understanding how to win boils down to understanding what pile sizes you want to leave your opponent with.

8. There are two incredibly powerful approaches to solving Nim. The first is to simplify. How could the game be easier? What if the pile had only one counter? From this place of almost absurd simplicity, we slowly raise the difficulty. What about two counters? Three counters?

9. The second approach is to organize the data in a coherent way. A table does this very nicely.

10. If student want to play three-player, keep in mind that we discourage it. Normally trying out different numbers of players is a great impulse. In Nim, it leads to spoilers, who can't win, but can choose who does win, which diffuses the mathematical tensions in the game.

11. Optional homework: have students teach 1-2 Nim to a friend or family member.