A Math Magic Trick

Topics: Addition, Subtraction, Multiplication, Division, Problem Solving
Materials: Scratch paper and pencil, or white boards, Cuisenaire rods (optional)
video launches (optional):
Part 1: **https://www.youtube.com/watch?v=n2RTjqHUXGQ**
Part 2: **https://www.youtube.com/watch?v=HMSmt5yOl6E**
Duration: 1 - 2 lessons

How does it come out the same every time?

What we love about a Math Magic Trick
This deceptively simple trick can differentiate broadly. Some students can use it to practice arithmetic or get practice with arithmetic for whole numbers, or for harder topics like say subtracting negative numbers or dividing fractions.

The Launch
The teacher explains the trick to the class. Every student follows the instructions:

- Pick a whole number between 1 and 10.
- Add 2.
- Multiply by 2.
- Subtract 2.
- Divide by 2.
- Subtract your original number.

Everyone's final answer will be 1 (assuming they didn't make any arithmetic mistakes).

Big Question 1. Will this work for number that aren't whole numbers between 1 and 10? What numbers will it work for?

The Work
For most students, trying to break this pattern is a great challenge. They can try big numbers, negative numbers, decimals, fractions, or other numbers on their own or with help from the teacher. Lo and behold, everything they try will work out.

The teacher can collect conjectures about what numbers won't come out to 1 when you apply the rules of the trick. Keep checking in to challenge students with other students' ideas, or help out with arithmetic that might be a bit beyond the student comfort level.

Prompts and Questions
- What's a number you think would break the pattern?
- Do you think 17 will work? Let's try it together.

- What's a number that might not work? Let's try to write some guesses down, so you have a few to try.
- Have you tried really large numbers? What about 40,156?
- Do you think negative numbers will work? What about -5?
- Do you think fractions will work? What about $\frac{1}{2}$? What about $3\frac{2}{3}$?
- Do decimals work? What about 6.7?
- What's the weirdest number you can imagine trying? What about π?
- Why do you think it will always work? Is it just because it has so far, or do you actually have a reason based on what we're doing?
- I wonder if we could use a bar model/algebra to help.
- (for students who have solved the problem completely) What if you change all the 2s to 3s? Will it still work?

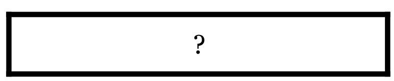
The Wrap

After they'd done a multitude of examples, we have the next question:

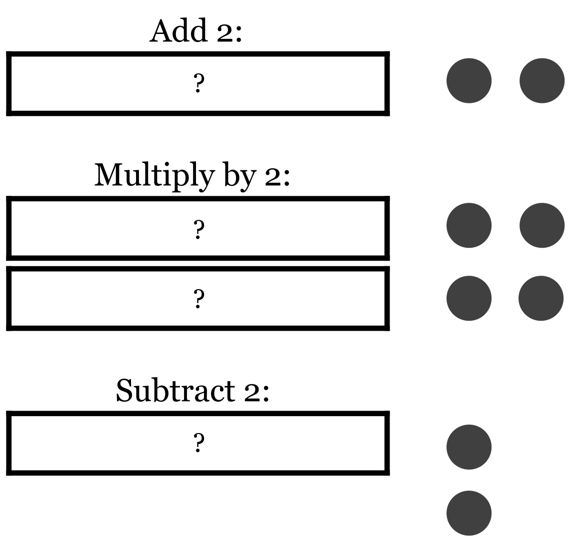
Big Question 2. Why does this always work?

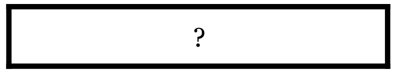
Taken at face value, this seems like an impossibly hard question. To show it always works, you need to test the trick for *every* number, and that means an infinite amount of work. How can you prove things like this are *always* true?

Depending on time, you may choose to let this question hang and pick it up when you return to this topic in a future lesson. You can also use a bar model or Cuisenaire rods to solve it. Choose a Cuisenaire rod or draw a bar model and assume this is a number, except we don't know what number it is yet.

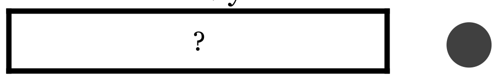


Apply the trick to the missing number, using counters to stand in for units.





Divide by 2:



Subtract your original number:

And here's the power of algebraic thinking made visible! No matter what that original Cuisenaire rod was worth, you always end up with 1 at the end. By leaving it at a question mark, we've actually managed to check every number at once!

This same work can be accomplished using algebra. For students who are not yet comfortable with algebra, this magic trick offers another opportunity to grasp the idea. For example, a common breakthrough is to try the trick with pi as the starting number. Since, as many students know, the digits of pi go on forever without repeating, we can't really write it out. But maybe we can just use the letter! This gives us:

π (pick a number)
$\pi + 2$ (add two)
$2\pi + 4$ (multiply by 2… this step can be a little tricky to those who don't know algebra)
$2\pi + 2$ (subtract 2)
$\pi + 1$ (divide by 2)
1 (subtract your original number)

Note: Video on this transition available at **https://youtu.be/HMSmt5yOl6E**

But here's the big idea of algebra: we didn't need to know anything about that π symbol to find out that the answer was 1. In fact, π could have stood for *any* number, and the answer would have been the same. Wait… if it could have stood for any number that the trick would still have given the answer 1, then the trick works for any number! This is actually an algebraic proof, and it gives an argument that the trick works. Always.

It may be nice to mention that folks will commonly use x rather than π.

x (pick a number)
$x + 2$ (add two)
$2x + 4$ (multiply by 2… this step can be a little tricky to those who don't know algebra)
$2x + 2$ (subtract 2)
$x + 1$ (divide by 2)
1 (subtract your original number)

From here, generalizing the trick takes us into natural extensions, which can be stand alone lessons for the future, or simply open questions for students to mull over on their own.

Big Question 3. What happens if you tinker with the trick?

A natural way to tinker is to double all the numbers.

Variation
Pick a number.
Add 4.
Multiply by 4.
Subtract 4.
Divide by 4.
Subtract your original number.

Do you always get the same answer?

Of course, students may have other ideas of how to tinker. Figuring out the relationship between what's changed in the trick and how the answer is affected is a great exploration, and students can go to town on it.

Tips for the Classroom

1. The main trick to this one is not to get students out of trouble too early. As long as they're confounded by the fact that the answer is always the same, you can keep goading them into trying another example (will it work for 5 halves? 137? -4.17?) and underlining how difficult the task of testing every number is. Don't rush students who aren't ready for the conceptual leap, and keep priming them to be ready for it when it comes.

2. Students will inevitably make arithmetic mistakes and think they've found a counterexample that doesn't come out to one. Have students check the work of others when they think they've found numbers that break the trick.

Maths Magic Trick

Pick a whole number between 1 and 10.

Add 2.

Multiply by 2.

Subtract 2.

Divide by 2.

Subtract your original number.

Your answer is…

Maths Magic Trick Challenge

Pick *any* number.

Add 2.

Multiply by 2.

Subtract 2.

Divide by 2.

Subtract your original number.

Find a number where the answer WON'T be 1.