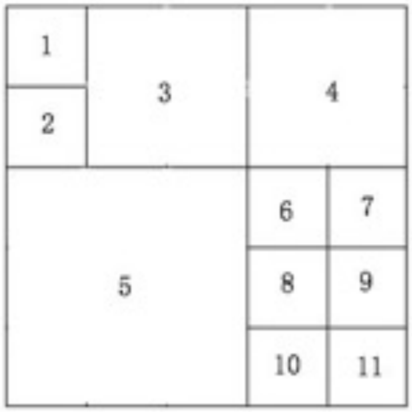
Squarable Numbers

Math Concepts: Geometry, Addition, geometric patterns, arithmetic patterns, modeling, linear equations
Materials: Graph paper, scratch paper, Video
Duration: 1-3 lessons



Why we love this lesson

This is a great problem, and kids can get working on it right away. Help them by giving them graph paper, and by getting them to circle the squarable numbers as they build diagrams on their paper: they build momentum immediately.

What's also great is that each number really corresponds to a student's way of cutting a square into squares, so that when students start finding patterns, those are really models that reflect a strategy at work. There are some powerful ideas at play here, and the student can get at them in a way that's motivated and meaningful.

The Launch

Use the accompanying video as an option launch.

Explain the premise of the problem. This is best demonstrated with an example. Here is one way to build a square from 11 smaller squares. Since we can cut a square into 11 smaller squares (of any size), we call the number 11 "squarable." In general, the number n is "squarable" if we can build a square out of precisely n smaller squares (of any size) with no leftover space.

Prompts and Questions

These first two are the guiding questions for this exploration.

- Which numbers are squarable?
- Is there a simple way to tell if a number is squarable or not squarable?

These are good prompts to use with students as they work.

- Which numbers do you conjecture are squarable?
- Which numbers do you think aren't squarable?
- Do you have a strategy to find squarable numbers?
- If so, can you describe it?
- It looks like you have a method of finding certain types of squarable numbers. Could you describe which numbers your method works for, (i.e., using an equation or some other description)?

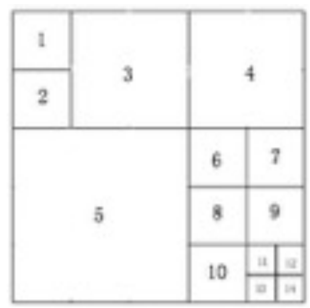
The Work

Part 1: Graph paper is helpful. To get students started, I like to start them with the numbers from 1 to 25. Every time you come up with a way to break a square into some smaller number of squares, we can circle that number. This way, every experiment can give us a productive outcome, which is an encouraging way to begin.

1 2 3 4 5 6 7 8 9 10 11 12 13

14 15 16 17 18 19 20 21 22 23 24 25

Part 2: After students start getting good at breaking up squares using trial and error, we want to encourage them to use more strategies. Are there any patterns or strategies that help you determine whether a number is squarable or not? Hopefully, students will come up with a few different tactics. Here's a good example of a powerful strategy, though:



- When you cut a single square into four, you effectively add 3 squares to the total. So knowing that 11 is squarable actually lets us determine that 14 is squarable as well: just cut one of the squares into four equal pieces.

Part 3: Especially for older students, strategies should give them a way to work upward to larger numbers. If you give them some specific challenges (Is 26 squarable? 31? 99? 1000? 1001?) , they can use their strategies to help them determine what works and what doesn't. For example, I know that I can add 3 to any number on my list of squarable numbers, and I also know that 11 is squarable. So I can add up by threes: 11, 14, 17, 20, 23, 26. Therefore, 26 is squarable, and I could draw a picture to prove it if necessary. To get to larger numbers, kids will need to come up with more sophisticated counting techniques. Division, for example.

- If appropriate, as we can extend the logic to say that 17, 20, 23, 26, etc. are all squarable numbers, we have, in essence, we've found a "family" of squarable numbers. If students are ready, they could write an equation (or just a description) of these numbers, i.e.:
  "Numbers that are one less than a multiple of 3 (starting from 11) are squarable."
  "Numbers of the form 3n - 1 are squarable, for n≥4."

Part 4: This probably won't happen in just one lesson, or even two, but it's possible to figure out all the numbers that are squarable. See if you and your kids can figure it out. If you can't, then it can be a longer-standing challenge to inspire more concerted work in the future.

The Wrap

On the first day, you can wrap by letting students share their strategies and arguments, and seeing what numbers are still question marks. What are the conjectures? Which conjectures have been broken?

If you pursue the question to subsequent days, you can help students tie together an argument about which numbers are actually squarable and which are not.

Tips for the Classroom

1. Encourage the use of graph paper!
2. Remember to encourage students to stake claims! They should have conjectures, and be disproving other conjectures.
3. Trying to show specific numbers are squarable is much harder than building examples and tracking what you got. If students are keeping a good list of which numbers they have shown are squarable, they'll have the successes under their belt to motivate them to look at harder numbers.
4. If there are particularly tricky numbers (i.e., "no one has proved 13 is squarable yet") you can put a "bounty" on that number, and motivate the students to go after it especially.
5. Going to larger numbers will motivate a shift from the concrete work of drawing the pictures to the abstract work of describing and using the strategies for building squarable numbers.
6. Students may surprise you with the diversity of their ideas for strategies. Be open to what they come up with, even if it is unexpected!
7. Spoiler: It turns out all numbers are squarable except 2, 3, and 5. Proving 5 is not squarable is a fairly subtle geometric exercise.